

To Appear in the Astrophysical Journal, v612, Sept. 10, 2004

## Direct Determination of the Kinematics of the Universe and Properties of the Dark Energy as Functions of Redshift

Ruth A. Daly

*Department of Physics, Berks-Lehigh Valley College, Penn State University, Reading, PA 19610*

rdaly@psu.edu

and

S. G. Djorgovski

*Division of Physics, Mathematics, and Astronomy, California Institute of Technology, MS 105-24, Pasadena, CA 91125*

george@astro.caltech.edu

### ABSTRACT

Understanding of the nature of dark energy, which appears to drive the expansion of the universe, is one of the central problems of physical cosmology today. In an earlier paper [Daly & Djorgovski (2003)] we proposed a novel method to determine the expansion rate  $E(z)$  and the deceleration parameter  $q(z)$  in a largely model-independent way, directly from the data on coordinate distances  $y(z)$ . Here we expand this methodology to include measurements of the pressure of dark energy  $p(z)$ , its normalized energy density fraction  $f(z)$ , and the equation of state parameter  $w(z)$ . We then apply this methodology to a new, combined data set of distances to supernovae and radio galaxies. In evaluating  $E(z)$  and  $q(z)$ , we make only the assumptions that the FRW metric applies, and that the universe is spatially flat (an assumption strongly supported by modern CMBR measurements). The determinations of  $E(z)$  and  $q(z)$  are independent of any theory of gravity. For evaluations of  $p(z)$ ,  $f(z)$  and  $w(z)$ , a theory of gravity must be adopted, and General Relativity is assumed here. No a priori assumptions regarding the properties or redshift evolution of the dark energy are needed. We obtain trends for  $y(z)$  and  $E(z)$  which are fully consistent with the standard Friedmann-Lemaître concordance cosmology with  $\Omega_0 = 0.3$  and  $\Lambda_0 = 0.7$ .

The measured trend for  $q(z)$  deviates systematically from the predictions of this model on a  $\sim 1 - 2 \sigma$  level, but may be consistent for smaller values of  $\Lambda_0$ . We confirm our previous result that the universe transitions from acceleration to deceleration at a redshift  $z_T \approx 0.4$ . The trends for  $p(z)$ ,  $f(z)$ , and  $w(z)$  are consistent with being constant at least out to  $z \sim 0.3 - 0.5$ , and broadly consistent with being constant out to higher redshifts, but with large uncertainties. For the present values of these parameters we obtain:  $E_0 = 0.97 \pm 0.03$ ,  $q_0 = -0.35 \pm 0.15$ ,  $p_0 = -0.6 \pm 0.15$ ,  $f_0 = -0.62 - (\Omega_0 - 0.3) \pm 0.05$ , and  $w_0 = -0.9 - \epsilon(\Omega_0 - 0.3) \pm 0.1$ , where  $\Omega_0$  is the density parameter for nonrelativistic matter, and  $\epsilon \approx 1.5 \pm 0.1$ . We note that in the standard Friedmann-Lemaitre models  $p_0 = -\Lambda_0$ , and thus we can measure the value of the cosmological constant directly, and obtain results in agreement with other contemporary results.

*Subject headings:* cosmological parameters - cosmology: observations - cosmology: theory - dark matter - equation of state

## 1. Introduction

Observations of supernovae (Riess et al. 1998; Perlmutter et al. 1999; Tonry et al. 2003; Knop et al. 2003; Barris et al. 2004; Riess et al. 2004) indicate that the universe is accelerating in its expansion. Precision measurements of cosmological parameters from CMBR experiments confirm this remarkable finding (e.g., Bennett et al. 2003, Spergel et al. 2003, and references therein). Results similar to those obtained using supernovae are also obtained using radio galaxies (Guerra & Daly 1998; Guerra, Daly, & Wan 2000; Daly & Guerra 2002; Podariu et al. 2003). The acceleration of the universe at the present epoch is one of the key results of modern cosmology, with potentially significant implications for fundamental physics as well. The nature of the “dark energy,” which apparently drives the cosmic acceleration, is unknown and it is crucially important to extract information about it from the data in a manner that is as direct and model-independent as possible.

In Daly & Djorgovski (2003; hereafter paper I), we showed how the data could be used to study the dimensionless expansion rate of the universe,  $E(z)$ , and the deceleration parameter of the universe,  $q(z)$  directly from combinations of the first and second derivatives of the coordinate distance. These determinations only depend upon the Friedmann-Robertson-Walker metric, and an assumption of spatially flat geometry, which is now very well established by the CMBR experiments. The evaluations do not require the specification of anything else, including a theory of gravity, and thus are direct and model-independent.

The use of model-independent methods to derive information about the dark energy are also discussed, for example, by Huterer & Turner (1999, 2001), Saini et al. (2000), Tegmark (2002), Sahni et al. (2003), Huterer & Starkman (2003), Wang and Freese (2004), Wang & Tegmark (2004), Wang et al. (2004), and Daly & Djorgovski (2004). The work of Huterer & Turner focuses on determinations on  $w(z)$ , as does that of Huterer & Starkman (2003). Wang & Freese (2004) focus on the determination of the energy density of the dark energy, and use an approach that is complementary to that used here, by integrating over shells in redshift space to obtain the energy density as a function of redshift, while we differentiate the data to obtain this function. The approach taken by most authors to extract the redshift behavior of the dark energy is to integrate over an assumed functional form of the redshift evolution of the dark energy, having first adopted a theory of gravity (e.g. Starobinsky 1998; Huterer & Turner 1999, 2001; Saini et al. 2000; Chiba & Nakamura 2000; Maor, Brustein, & Steinhardt 2001; Golaith et al. 2001; Wang & Garnavich 2001; Astier 2001; Gerke & Efstathiou 2002; Weller & Albrecht 2002; Padmanabhan & Choudhury 2002; Tegmark 2002; Huterer & Starkman 2003; Sahni et al. 2003; Alam et al. 2003; Wang & Freese 2004; Wang et al. 2004; Wang & Tegmark 2004; Nessier & Perivolaropoulos 2004; Gong 2004; Zhu, Fujimoto, & He 2004; Elgaroy & Multamaki 2004; Huterer & Cooray 2004; Alam, Sahni, & Starobinsky 2004). However, it can be difficult to extract information about the redshift behavior of the dark energy using these “integral” approaches (Maor, Brustein, & Steinhardt 2001; and Barger & Marfatia 2001). Thus, we continue to follow the complementary approach of differentiating the data, as described in paper I.

Here, the approach presented in paper I is taken a step further, to obtain the pressure, energy density, and equation of state of the dark energy directly from combinations of the first and second derivatives of the coordinate distance with respect to redshift. This approach is complementary to the standard approach of assuming a theory of gravity, assuming a parameterization for the dark energy and its redshift evolution, and obtaining the best fit model parameters.

We apply this methodology on an improved set of distances to supernovae (SNe) from Riess et al. (2004), supplemented with the data on high-redshift radio galaxies (RGs) from paper I.

## 2. Theory

This work builds upon paper I, and we refer the reader to it for more details and discussion. It is well known that the dimensionless expansion rate  $E(z)$  can be written as the derivative of the dimensionless coordinate distances  $y(z)$  (e.g. Weinberg 1972; Peebles

1993; Peebles & Ratra 2003); the expression is particularly simple when the space curvature term is equal to zero. In this case,

$$\left(\frac{\dot{a}}{a}\right) H_0^{-1} \equiv E(z) = (dy/dz)^{-1} , \quad (1)$$

where  $a$  is the cosmic scale factor, and  $H_0 = (\dot{a}/a)|_0$  evaluated at a redshift of zero is Hubble's constant. This representation follows directly from the Friedman-Robertson-Walker line element, and does not require the use of a theory of gravity. Similarly, it is shown in paper I that the dimensionless deceleration parameter

$$-\left(\frac{\ddot{a}a}{\dot{a}^2}\right) \equiv q(z) = -[1 + (1+z)(dy/dz)^{-1} d^2y/dz^2] \quad (2)$$

also follows directly from the FRW line element, and does not rely upon a theory of gravity. Thus, measurements of the dimensionless coordinate distance to sources at different redshifts can be used to determine  $dy/dz$  and  $d^2y/dz^2$ , which can then be used to determine  $E(z)$  and  $q(z)$ .

In addition, if a theory of gravity is specified, the measurements of  $dy/dz$  and  $d^2y/dz^2$  can be used to determine the pressure, energy density, and equation of state of the dark energy as functions of redshift. Thus, we can use the data to determine these functions directly, which provides an approach that is complementary to the standard one of assuming a functional form a priori, and then fitting the parameters of the chosen function. To determine the pressure, energy density, and equation of state of the dark energy as functions of redshift, the theory of gravity adopted is General Relativity.

In a spatially flat, homogeneous, isotropic universe with non-relativistic matter and dark energy Einstein's equations are

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3} (\rho_m + \rho_{DE} + 3P_{DE}) \quad (3)$$

and

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} (\rho_m + \rho_{DE}) , \quad (4)$$

where  $\rho_m$  is the mean mass-energy density of non-relativistic matter,  $\rho_{DE}$  is the mean mass-energy density of the dark energy, and  $P_{DE}$  is the pressure of the dark energy. Combining these equations, we find  $(\ddot{a}/a) = -0.5[(\dot{a}/a)^2 + (8\pi G) P_{DE}]$ .

Using the standard definition of the critical density at the present epoch  $\rho_{oc} = 3H_0^2/(8\pi G)$ , it is easy to show that

$$p(z) \equiv \left(\frac{P_{DE}(z)}{\rho_{oc}}\right) = \left(\frac{E^2(z)}{3}\right) [2q(z) - 1] . \quad (5)$$

Equations (1) and (2) can be used to obtain the pressure of the dark energy as a function of redshift

$$p(z) = -(dy/dz)^{-2} [1 + (2/3) (1+z) (dy/dz)^{-1} (d^2y/dz^2)] . \quad (6)$$

Thus, the pressure of the dark energy can be determined directly from measurements under the same assumptions as above. Moreover, for the standard Friedmann-Lemaitre models, it can be shown that  $p = -\Omega_\Lambda$ , giving us a way to measure the value of the cosmological constant directly.

Similarly, the energy density of the dark energy can be obtained directly from the data using equations (1) and (4):

$$f(z) \equiv \left( \frac{\rho_{DE}(z)}{\rho_{oc}} \right) = (dy/dz)^{-2} - \Omega_0(1+z)^3 , \quad (7)$$

where  $\Omega_0 = \rho_{om}/\rho_{oc}$  is the fractional contribution of non-relativistic matter to the total critical density at zero redshift, and it is assumed that this non-relativistic matter evolves as  $(1+z)^3$ .

The equation of state  $w(z)$  is defined to be the ratio of the pressure of the dark energy to its energy-density  $w(z) \equiv P_{DE}(z)/\rho_{DE}(z)$ . Combining equations (6) and (7), it is easy to show that

$$w(z) = -[1 + (2/3) (1+z) (dy/dz)^{-1} (d^2y/dz^2)]/[1 - (dy/dz)^2 \Omega_0 (1+z)^3] . \quad (8)$$

### 3. Data Analysis and Results

Our method is based on a robust numerical differentiation of data on coordinate distances  $y(z)$ , which is described in detail in paper I. One of the advantages of our method is that distances from different types of measurements (e.g., SNe standard candles, and RG standard rulers) can be combined, separating the astrophysical questions (how standard are these sources, what are the selection effects, etc.) from analyses dealing with pure geometry and kinematics.

Two data samples are included in this study: the RG sample presented and described by Guerra, Daly, & Wan (2000), Daly & Guerra (2002), Podariu et al. (2003), and in paper I, and the latest cosmological SNe sample from Riess et al. (2004). The RG sample consists of 20 RGs with redshifts between zero and 1.8 (Guerra, Daly, & Wan 2000). The SNe sample that we use here consists of the “gold” SNe, with redshifts between zero and 1.7 (Riess et al. 2004). We refer to the original papers for the description of the measurements and other pertinent information.

The dimensionless coordinate distances  $y(z)$  to RGs were determined in paper I for normalizations obtained using RGs alone (referred to as  $y_s$ ), and obtained using a joint sample of RGs and SNe (referred to as  $y_j$ ). The current SNe sample is used to obtain new values of  $y_j$  by using them to determine a new normalization for the RGs, and these are listed in Table 1. These values are nearly identical to those in paper I.

The dimensionless coordinate distances to SNe are listed in Table 2. To determine these from the distance moduli published by Riess et al. (2004), the value of  $H_0$  adopted by Riess et al. (2004) must be known. This was not given explicitly in the Riess et al. paper, as it was not needed for their analysis. Since we essentially need to remove the value and uncertainty of  $H_0$  put in by Riess et al. (2004), we determine the effective value of  $H_0$  applicable to that SNe sample by using the subsample of SNe with  $z < 0.1$ , where the expansion must be close to linear, and the Hubble relation  $H_0 = v(1+z)/d_L$  is valid. Using the luminosity distance  $d_L$  obtained directly from the distance moduli tabulated by Riess et al. (2004), we get  $H_0 = 66.4 \pm 0.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . This value is used simply to obtain the dimensionless coordinate distances  $y(z)$  from the published luminosity distances using the relation  $y(z) = (H_0/c)d_L(1+z)^{-1}$ , but it does not affect our analysis in any other way. It is not meant as a measurement of  $H_0$  per se, but just as an internally consistent scaling factor, and the error quoted above is just statistical, and does not include any other components due to calibrations, etc. The values of  $y(z)$  given in Tables 1 and 2 can then be easily converted to distances in parsec for any desired value of  $H_0$ .

We test for the consistency between the distance measurements from SNe and RGs in the redshift interval where they overlap (Figure 1). Reassuringly, we find no significant systematic offset, which indicates that the joint sample is sufficiently homogeneous for our purposes. We note that we repeated our analysis for the SN sample alone, and got essentially the same results, but with larger error bars at the high-redshift end, where the sample of SNe is still very sparse, and RGs provide valuable supplementary data. At the low redshifts, SNe dominate the results.

Our methodology is described in detail in paper I, which also includes extensive tests using simulated data. To summarize briefly, we perform a statistically robust numerical differentiation of the  $y(z)$  data, in order to obtain the first and second derivatives,  $dy/dz$  and  $d^2y/dz^2$  used in eqs. (1-8). While differentiation of noisy and sparse data is generally inadvisable, it is possible and may be useful if one keeps a careful track of the errors and other limitations posed by the data.

The procedure is based on properly weighted second-order least-squares fits at a closely spaced grid of redshift points, in a sliding redshift window, which is generally chosen to be sufficiently large ( $\Delta z = 0.4$  or  $0.6$ ) to have enough data points for meaningful measurements

of the 3 fit coefficients. The fit coefficients and their errors essentially correspond to the best fit values for  $y(z)$ ,  $dy/dz$  and  $d^2y/dz^2$ . We are effectively doing a Taylor series expansion for the expansion law as a function of redshift. Statistical errors, including all covariance terms, are propagated in the standard manner. While the large values of  $\Delta z$  are needed in order to obtain stable fits, that also means that there are very few independent intervals: we are essentially mapping the trends, rather than to try to bin the data. We find that the derived mean trends for all quantities of interest described below do not depend significantly on the value of  $\Delta z$  used, i.e., the results are robust with respect to this parameter. However, the statistical errors increase dramatically for lower values of  $\Delta z$ , due to the smaller numbers of enclosed data points.

While the fitting procedure generates statistically rigorous errors at every point, that does not include any effects of the uneven data sampling and sample variance (see the discussion in paper I). The  $1\text{-}\sigma$  error intervals plotted in the figures reflect only the statistical errors. The apparent “bumps and wiggles” are presumably indicative of the sparse sampling, especially at higher redshifts. Any systematic errors in  $y(z)$  measurements which may be present in the data are also absorbed there. Thus, one should not believe any such features in the plots, but only look at the global trends. We also regard the values for all derived quantities at lower redshifts to be fairly reliable, since the data are the best and the sampling is densest as  $z \rightarrow 0$ .

As in paper I, we perform a test of the procedure using a simulated data set which mimics the anticipated SN measurements from the SNAP/JDEM satellite (see <http://snap.lbl.gov>), with a known assumed cosmology, namely the standard Friedmann-Lemaitre model with  $\Omega_0 = 0.3$  and  $\Lambda_0 = 0.7$  (see paper I for more details on this simulated data set). The results for the dark energy parameters as functions of redshift are shown in Figure 2. We see that our method can recover robustly the assumed parameters, at least out to  $z \approx 0.9$ . Reassured by this test, we turn to the analysis of actual data.

We do not endeavor here to examine or advocate the primary measurements of distances we use in our analysis; that was done in the original papers from which they came. Our purpose here is to illustrate the methodology and seek some early hints about the possible cosmological trends in the data, assuming that the data are sound. Better and larger data sets in the future can be explored using this methodology with a much greater potential.

Figure 3 shows the data from the combined RG+SN (gold) sample, and the representative fits for  $y(z)$ , for window function widths  $\Delta z$  of 0.4 and 0.6. Figure 4 shows the corresponding results for the dimensionless expansion rate,  $E(z)$ . We obtain the present value of  $E_0 = 0.97 \pm 0.03$ . Both trends,  $y(z)$  and  $E(z)$  are fully consistent with the standard concordance model, which assumes  $w = -1$ ,  $\Omega_0 = 0.3$ , and  $\Lambda_0 = 0.7$ .

Figure 5 shows the trend for the deceleration parameter  $q(z)$ . This is an update of our result from paper I, which we believe was the first direct demonstration of the transition from a decelerating to an accelerating universe. This was subsequently seen by Riess et al. (2004), and is further confirmed here, and by Alam, Sahni, & Starobinsky (2004). We see a clear trend of an increase in  $q(z)$  with redshift out to  $z \sim 0.6$ , but the fits become noisy and unreliable beyond that, due to the still limited number of data points at higher redshifts. The present value is estimated at  $q_0 = -0.35 \pm 0.15$ .

The zero crossing is seen at  $z_T \approx 0.4$ ; specifically, for the window function with  $\Delta z = 0.6$ , it is  $z_T = 0.35 \pm 0.07$ . While the value of  $z_T$  does not depend significantly on the value of  $\Delta z$  used, the size of the uncertainty does, and we are reluctant to quote one particular case. While the lower limit is relatively robust, the upper bound is very uncertain due to the sparse sampling at higher redshifts. We note that in the simple Friedmann-Lemaître models  $z_T = (2\Omega_\Lambda/\Omega_0)^{1/3} - 1$ . For the standard concordance model with  $w = -1$ ,  $\Omega_0 = 0.3$ , and  $\Omega_\Lambda = 0.7$ , we would expect  $z_T = 0.67$ . If  $z_T = 0.35$ , then the implied value is  $\Omega_\Lambda = 0.55$  for a  $k = 0$  model. Indeed, the evaluated trend for  $q(z)$  is closer to the  $\Omega_\Lambda = 0.5$  model than to the  $\Omega_\Lambda = 0.7$  case, which seems systematically low at a 1 to 2  $\sigma$  level (statistical errors only). However, given the limitations presented by the available data sample, we are unsure about the significance of this effect.

For the subsequent measurements, the assumption that GR is the correct theory of gravity is made (see the previous section).

Equation (6) is used to obtain the pressure of the dark energy as a function of redshift, and the results are shown in Figure 6, for a window function with  $\Delta z = 0.6$ . The present value is  $p_0 = -0.6 \pm 0.15$ . The results are consistent with the pressure remaining constant to  $z \sim 0.5$ , and possibly beyond; the strong fluctuations at higher redshifts, due to a sparser sampling of data, preclude any stronger statements at this point.

Note that  $\rho_{DE0} = P_{DE0}/w_0$  so the value of  $p_0$  can be used to determine  $w_0$  if  $\rho_{DE0}$  is known, or vice versa; for  $\Omega_{DE0} = 0.7$ , our determination of  $p_0$  implies  $w_0 = -0.86 \pm 0.21$ ; or, for  $w_0 = -1$ , our determination of  $p_0$  implies  $\Omega_{DE0} = 0.6 \pm 0.15$ , which is fully consistent with other measurements of the cosmological constant and our own estimate from the  $z_T$  given above.

Equation (7) is used to obtain the energy density of the dark energy as a function of redshift, as shown in Figure 7 for the window function width  $\Delta z = 0.6$ , assuming the mean mass density in non-relativistic matter at zero redshift is  $\Omega_0 = 0.3$  (implementing different choices for  $\Omega_0$  is trivial). The present value is  $f(z) = 0.62 \pm 0.05$ . The data are consistent with constant mean dark energy density out to  $z \sim 0.5$  and possibly beyond.



Equation (8) is used to study the equation of state parameter  $w(z)$  as a function of redshift and the results are shown in Figure 8. The present value  $w_0 = -0.9 \pm 0.1$  is fully consistent with the interpretation of the dark energy as a cosmological constant ( $w = -1$ ). However, the trend out to  $z \sim 0.6$  is intriguing. We are uncertain at this point whether this is simply due to a sampling-induced fluctuation (as is obviously the case at higher redshifts), or whether there may be a real evolution of  $w(z)$ . Clearly, to invoke the standard cosmological truism, more data are needed.

In all of our analysis, we have also considered different samples and subsamples of data, such as including a sample of just the “silver” and “gold” SNe (Riess et al. 2004), the “gold” SNe alone, and the sample of RGs plus “silver” and “gold” SNe; the results are effectively the same as those shown here. However, we note that since the SNe dominate the joint sample, all of our results are just as vulnerable to any hidden systematic errors which may be present in the data as the more traditional analysis presented by Riess et al.

#### 4. Summary

We expanded and used the method developed in paper I on a new sample of coordinate distances to SNe and RGs, to evaluate the trends of the expansion rate  $E(z)$ , deceleration parameter  $q(z)$ , pressure of the dark energy  $p(z)$ , its fractional energy density  $f(z)$ , and its equation of state parameter  $w(z)$ , as functions of redshift. We make an assumption that the FRW metric is valid, and the observationally supported assumption of the spatially flat universe. This enables us to derive the trends for  $E(z)$  and  $q(z)$  which are otherwise model-independent, and thus can help discriminate at least some proposed models of the dark energy. By assuming that the standard GR is the correct theory of gravity on cosmological scales, we can also produce trends of  $p(z)$ ,  $f(z)$  and  $w(z)$ , without any additional assumptions about the nature of dark energy. These trends may be also used to discriminate between different physical models of the dark energy.

We find that the data are generally, but perhaps not entirely consistent with the standard Friedmann-Lemaitre concordance cosmology with  $w = -1$ ,  $\Omega_0 = 0.3$ , and  $\Lambda_0 = 0.7$ , although somewhat lower values of  $\Lambda_0$  may be preferred.

We confirm the result paper I and that of Riess et al. (2004), that there is a clear increase  $q(z)$  with redshift, with the present value  $q_0 = 0.35 \pm 0.1$ , and the transition from decelerating to accelerating universe at  $z_T \approx 0.4$ .

Functions  $p(z)$ ,  $f(z)$ , and  $w(z)$  are consistent with being constant at least out to  $z \sim 0.5$ , and possibly beyond; the existing data are inadequate to constrain their evolution beyond

$z \sim 0.5$ , but there are some hints of increase with redshift for  $f(z)$  and  $w(z)$ .

At lower redshifts, the data are consistent with cosmological constant models. We obtain for the present values  $w_0 = -0.9 \pm 0.1$  and  $p_0 = -0.6 \pm 0.15$  ( $= -\Lambda_0$  for the Friedmann-Lemaitre models).

Even with the currently available data, these results represent new observational constraints for models of the dark energy. We believe that this methodology will prove increasingly useful in determining the nature and evolution of the dark energy as better and more extensive data sets become available. Clearly, this method works best when redshift space is densely sampled. Our current results suggest that redshift space is sufficiently sampled at redshifts less than about 0.4. More accurate results could be obtained by increasing the sampling of data points with redshifts greater than 0.4, particularly in the redshift range from 0.4 to 1.5.

It is a pleasure to thank Megan Donahue, Chris O’Dea, Saul Perlmutter, and Adam Riess for helpful discussions. This work was supported in part by the U. S. National Science Foundation under grants AST-0206002, and Penn State University (RAD), and by the Ajax Foundation (SGD). Finally, we acknowledge the great work and efforts of many observers who obtained the valuable data used in this study.

## REFERENCES

- Alam, U., Sahni, V., Saini, T. D., & Starobinsky, A. A. 2003, MNRAS, 344, 1057  
 Alam, U., Sahni, V., & Starobinsky, A. 2004, astro-ph/0403687  
 Astier, P. 2001, Phys. Lett. B, 500, 8  
 Barris, B. J., et al. 2004, ApJ, 602, 571  
 Barger, V., & Marfatia, D. 2001, Phys. Lett. B, 498,  
 Bennett, C., et al. (the WMAP team) 2003, ApJ, 583, 1  
 Chiba, T., & Nakamura, T. 2000, Phys. Rev. D, 62, 121301  
 Daly, R. A., & Djorgovski, S. 2003, ApJ, 597, 9  
 Daly, R. A., & Djorgovski, S. 2004, astro-ph/0405063  
 Daly, R. A., & Guerra, E. J. 2002, AJ, 124, 1831  
 Elgaroy, O., & Multamaki, T. 2004, astro-ph/0404402  
 Gerke, B. F., & Efstathiou 2002, MNRAS, 335, 33

- Goliath, M., Amanullah, T., Astier, P. Goobar, A., & Pain, R. 2001, *A&A*, 380, 6
- Gong, Y. 2004, *astro-ph/0401207*
- Guerra, E. J., & Daly, R. A. 1998, *ApJ*, 493, 536
- Guerra, E. J., Daly, R. A., & Wan, L. 2000, *ApJ*, 544, 659
- Huterer, D., & Cooray, A. 2004, *astro-ph/0404062*
- Huterer, D., & Starkman, G. 2003, *Phys. Rev. Lett.* 90, 031301
- Huterer, D., & Turner, M. S. 1999, *Phys. Rev. D*, 60, 081301
- Huterer, D., & Turner, M. S. 2001, *Phys. Rev. D*, 64, 123527
- Knop, R. A., et al. 2004, *ApJ*, in press (*astro-ph/0309368*)
- Maor, I., Brustein, R., & Steinhardt, P. 2001, *Phys. Rev. Lett.*, 86, 6
- Nessier, S., & Perivolaropoulos, L. 2004, *astro-ph/0401556*
- Padmanabhan, T., Choudhury, T. R. 2002, *MNRAS*, 344, 823
- Peebles, P. J. E. 1993, *Principles of Physical Cosmology*, Princeton University Press
- Peebles, P. J. E., & Ratra, B. 2003, *Rev. Mod. Phys.*, 75, 559
- Perlmutter et al. 1999, *ApJ*, 517, 565
- Podariu, S., Daly, R. A., Mory, M. P., & Ratra, B. 2003, *ApJ*, in press
- Riess, R. G., Strolger, L., Tonry, J., Casertano, S., Ferguson, H. G., Mobasher, B., Challis, P., Filippenko, A. V., Jha, S., Li, W., Chornock, R., Kirshner, R. P., Leibundgut, B., Dickinson, M., Livio, M., Giavalisco, M., Steidel, C. C., Benitez, N., & Tsvetanov, Z. 2004, *ApJ*, in press
- Riess, A. G. et al. 1998, *AJ*, 116, 1009
- Sahni, V., Saini, T. D., Starobinsky, A. A., & Alam, U. 2003, *J. Exp. Theor. Phys. Lett.*, 77, 201
- Saini, T., Raychaudhury, S., Sahni, V., & Starobinsky, A. A. 2000, *Phys. Rev. Lett.*, 85, 1162
- Spergel, D., et al. (the WMAP team) 2003, *ApJS*, 148, 175
- Tegmark, M. 2002, *Phys. Rev. D* 66, 103507
- Tonry, J. T. et al. 2004, *ApJ*, 594, 1
- Starobinsky, A., 1988, *JETP Lett.*, 68, 757
- Wang, Y., & Garnavich, P. 2001, *ApJ*, 552, 445

- Wang, Y., & Freese, K. 2004, astro-ph/0402208
- Wang, Y., Kostov, V., Freese, K., Frieman, J. A., & Gondolo, P. 2004, astro-ph/0402080
- Wang, Y., & Tegmark, M 2004, astro-ph/0403292
- Weinberg, S. 1972, Gravitation and Cosmology, John Wiley & Sons
- Weller, J., & Albrecht, A. 2002, Phys. Rev. D, 65, 103512
- Zhu, Z., Fujimoto, & He, X. 2004, astro-ph/0403228

Table 1. RG Dimensionless Coordinate Distances

Source	Redshift	$y$	$\sigma(y)$
3C405	0.0560	0.0556	0.0095
3C244.1	0.4300	0.4559	0.0700
3C330	0.5490	0.4019	0.0637
3C427.1	0.5720	0.3193	0.0488
3C337	0.6300	0.6094	0.0687
3C55	0.7200	0.5986	0.0678
3C247	0.7490	0.6255	0.0665
3C265	0.8110	0.6757	0.0787
3C325	0.8600	0.8180	0.1489
3C289	0.9670	0.6809	0.1030
3C268.1	0.9740	0.7679	0.1186
3C280	0.9960	0.7108	0.1073
3C356	1.0790	0.8284	0.1421
3C267	1.1440	0.7526	0.1206
3C194	1.1900	1.1412	0.1975
3C324	1.2100	0.9730	0.2350
3C437	1.4800	0.8211	0.1895
3C68.2	1.5750	1.4770	0.3690
3C322	1.6810	1.1406	0.2309
3C239	1.7900	1.2144	0.2376

Table 2. SNe Ia Dimensionless Coordinate Distances

Source	Redshift	$y$	$\sigma(y)$	sample
1990T	0.040	0.040	0.0035	gold
1990af	0.050	0.049	0.0048	gold
1990O	0.031	0.033	0.0030	gold
1991S	0.056	0.061	0.0050	gold
1991U	0.033	0.027	0.0025	gold
1992J	0.046	0.039	0.0038	gold
1992P	0.027	0.029	0.0027	gold
1992aq	0.101	0.112	0.0103	gold
1992ae	0.075	0.074	0.0065	gold
1992au	0.061	0.060	0.0061	gold
1992al	0.014	0.015	0.0017	gold
1992ag	0.026	0.022	0.0025	gold
1992bl	0.043	0.043	0.0038	gold
1992bh	0.045	0.052	0.0043	gold
1992bg	0.036	0.037	0.0032	gold
1992bk	0.058	0.056	0.0049	gold
1992bs	0.063	0.071	0.0062	gold
1992bc	0.019	0.021	0.0022	gold
1992bp	0.079	0.079	0.0066	gold
1992br	0.088	0.084	0.0108	gold
1992bo	0.018	0.019	0.0020	gold
1993B	0.071	0.074	0.0065	gold
1993H	0.025	0.023	0.0022	gold
1993O	0.052	0.057	0.0047	gold
1993ah	0.029	0.027	0.0027	gold
1993ac	0.049	0.051	0.0047	gold
1993ag	0.050	0.055	0.0048	gold
1993ae	0.018	0.016	0.0017	gold
1994B	0.089	0.102	0.0080	silver
1994C	0.051	0.045	0.0033	silver

Table 2—Continued

Source	Redshift	$y$	$\sigma(y)$	sample
1994M	0.024	0.023	0.0021	gold
1994Q	0.029	0.030	0.0026	gold
1994S	0.016	0.017	0.0019	gold
1994T	0.036	0.034	0.0031	gold
1995E	0.012	0.009	0.0011	silver
1995K	0.478	0.469	0.0497	gold
1995M	0.053	0.057	0.0039	silver
1995ap	0.230	0.220	0.0467	silver
1995ao	0.300	0.242	0.0668	silver
1995ae	0.067	0.067	0.0105	silver
1995az	0.450	0.407	0.0394	gold
1995ay	0.480	0.445	0.0410	gold
1995ax	0.615	0.509	0.0539	gold
1995aw	0.400	0.405	0.0354	gold
1995as	0.498	0.648	0.0716	silver
1995ar	0.465	0.551	0.0558	silver
1995ac	0.049	0.042	0.0039	gold
1995ak	0.022	0.019	0.0019	gold
1995ba	0.388	0.414	0.0362	gold
1995bd	0.015	0.014	0.0017	gold
1996C	0.028	0.033	0.0030	gold
1996E	0.425	0.340	0.0626	gold
1996H	0.620	0.572	0.0790	gold
1996I	0.570	0.514	0.0592	gold
1996J	0.300	0.271	0.0312	gold
1996K	0.380	0.407	0.0412	gold
1996R	0.160	0.125	0.0230	silver
1996T	0.240	0.244	0.0483	silver
1996U	0.430	0.453	0.0709	gold
1996V	0.025	0.025	0.0029	silver

Table 2—Continued

Source	Redshift	$y$	$\sigma(y)$	sample
1996ab	0.124	0.136	0.0138	gold
1996bo	0.017	0.013	0.0016	gold
1996bv	0.017	0.015	0.0016	gold
1996bl	0.035	0.037	0.0032	gold
1996cg	0.490	0.487	0.0426	silver
1996cm	0.450	0.501	0.0438	silver
1996cl	0.828	0.750	0.1589	gold
1996ci	0.495	0.417	0.0365	gold
1996cf	0.570	0.505	0.0442	silver
1997E	0.013	0.014	0.0017	gold
1997F	0.580	0.568	0.0549	gold
1997H	0.526	0.472	0.0391	gold
1997I	0.172	0.171	0.0142	gold
1997N	0.180	0.186	0.0154	gold
1997P	0.472	0.467	0.0408	gold
1997Q	0.430	0.387	0.0321	gold
1997R	0.657	0.602	0.0555	gold
1997Y	0.017	0.018	0.0019	gold
1997ai	0.450	0.401	0.0425	gold
1997ac	0.320	0.327	0.0271	gold
1997aj	0.581	0.470	0.0411	gold
1997aw	0.440	0.502	0.0925	gold
1997as	0.508	0.312	0.0503	gold
1997am	0.416	0.411	0.0360	gold
1997ap	0.830	0.712	0.0623	gold
1997af	0.579	0.523	0.0458	gold
1997bh	0.420	0.351	0.0371	gold
1997bb	0.518	0.537	0.0742	gold
1997bj	0.334	0.253	0.0350	gold
1997ck	0.970	0.753	0.1317	silver



Table 2—Continued

Source	Redshift	$y$	$\sigma(y)$	sample
1997cn	0.018	0.017	0.0020	gold
1997cj	0.500	0.521	0.0480	gold
1997ce	0.440	0.401	0.0350	gold
1997dg	0.030	0.036	0.0033	gold
1997do	0.010	0.012	0.0019	gold
1997ez	0.778	0.720	0.1160	gold
1997ek	0.860	0.761	0.1052	gold
1997eq	0.538	0.490	0.0406	gold
1997ff	1.755	1.025	0.1653	gold
1998I	0.886	0.448	0.1672	gold
1998J	0.828	0.638	0.1793	gold
1998M	0.630	0.454	0.0502	gold
1998V	0.017	0.017	0.0018	gold
1998ac	0.460	0.352	0.0649	gold
1998ay	0.638	0.618	0.1024	silver
1998bi	0.740	0.595	0.0822	gold
1998be	0.644	0.484	0.0580	silver
1998ba	0.430	0.459	0.0528	gold
1998bp	0.010	0.010	0.0014	gold
1998co	0.017	0.019	0.0021	gold
1998cs	0.033	0.035	0.0031	gold
1998dx	0.053	0.052	0.0043	gold
1998ef	0.017	0.015	0.0016	gold
1998eg	0.023	0.026	0.0024	gold
1999Q	0.460	0.493	0.0613	gold
1999U	0.500	0.524	0.0458	gold
1999X	0.026	0.026	0.0024	gold
1999aa	0.016	0.018	0.0020	gold
1999cc	0.032	0.032	0.0028	gold
1999cp	0.010	0.011	0.0016	gold

Table 2—Continued

Source	Redshift	$y$	$\sigma(y)$	sample
1999da	0.012	0.014	0.0021	silver
1999dk	0.014	0.017	0.0020	gold
1999dq	0.014	0.012	0.0015	gold
1999ef	0.038	0.046	0.0038	gold
1999fw	0.278	0.274	0.0518	gold
1999fk	1.056	0.762	0.0807	gold
1999fm	0.949	0.713	0.0821	gold
1999fj	0.815	0.689	0.1047	gold
1999ff	0.455	0.437	0.0563	gold
1999fv	1.190	0.696	0.1090	gold
1999fh	0.369	0.341	0.0487	silver
1999fn	0.477	0.448	0.0434	gold
1999gp	0.026	0.029	0.0026	gold
2000B	0.019	0.018	0.0019	gold
2000bk	0.027	0.025	0.0025	gold
2000cf	0.036	0.041	0.0034	gold
2000cn	0.023	0.023	0.0022	gold
2000ce	0.016	0.017	0.0018	silver
2000dk	0.016	0.017	0.0018	gold
2000dz	0.500	0.524	0.0579	gold
2000eh	0.490	0.451	0.0519	gold
2000ee	0.470	0.532	0.0563	gold
2000eg	0.540	0.354	0.0669	gold
2000ea	0.420	0.224	0.0330	silver
2000ec	0.470	0.539	0.0521	gold
2000fr	0.543	0.493	0.0431	gold
2000fa	0.022	0.022	0.0022	gold
2001V	0.016	0.015	0.0015	gold
2001fs	0.873	0.665	0.1163	gold
2001fo	0.771	0.526	0.0412	gold

Table 2—Continued

Source	Redshift	$y$	$\sigma(y)$	sample
2001hy	0.811	0.761	0.1226	gold
2001hx	0.798	0.735	0.1049	gold
2001hs	0.832	0.620	0.0827	gold
2001hu	0.882	0.709	0.0979	gold
2001iw	0.340	0.229	0.0285	gold
2001iv	0.397	0.239	0.0330	gold
2001iy	0.570	0.531	0.0758	gold
2001ix	0.710	0.527	0.0777	gold
2001jp	0.528	0.519	0.0597	gold
2001jh	0.884	0.824	0.0721	gold
2001jb	0.698	0.604	0.0890	silver
2001jf	0.815	0.802	0.1034	gold
2001jm	0.977	0.678	0.0811	gold
2001kd	0.935	0.718	0.1257	silver
2002P	0.719	0.567	0.0679	silver
2002ab	0.422	0.395	0.0309	silver
2002ad	0.514	0.439	0.0546	silver
2002dc	0.475	0.402	0.0352	gold
2002dd	0.950	0.736	0.0882	gold
2002fw	1.300	1.090	0.0953	gold
2002fx	1.400	0.961	0.1992	silver
2002hr	0.526	0.580	0.0721	gold
2002hp	1.305	0.836	0.0847	gold
2002kc	0.216	0.212	0.0176	silver
2002kd	0.735	0.529	0.0463	gold
2002ki	1.140	0.961	0.1327	gold
2003az	1.265	1.071	0.0987	gold
2003ak	1.551	0.996	0.1009	gold
2003bd	0.670	0.576	0.0743	gold
2003be	0.640	0.555	0.0537	gold

Table 2—Continued

Source	Redshift	$y$	$\sigma(y)$	sample
2003dy	1.340	0.968	0.1114	gold
2003es	0.954	0.813	0.1161	gold
2003eq	0.839	0.712	0.0721	gold
2003eb	0.899	0.623	0.0717	gold
2003lv	0.940	0.678	0.0624	gold

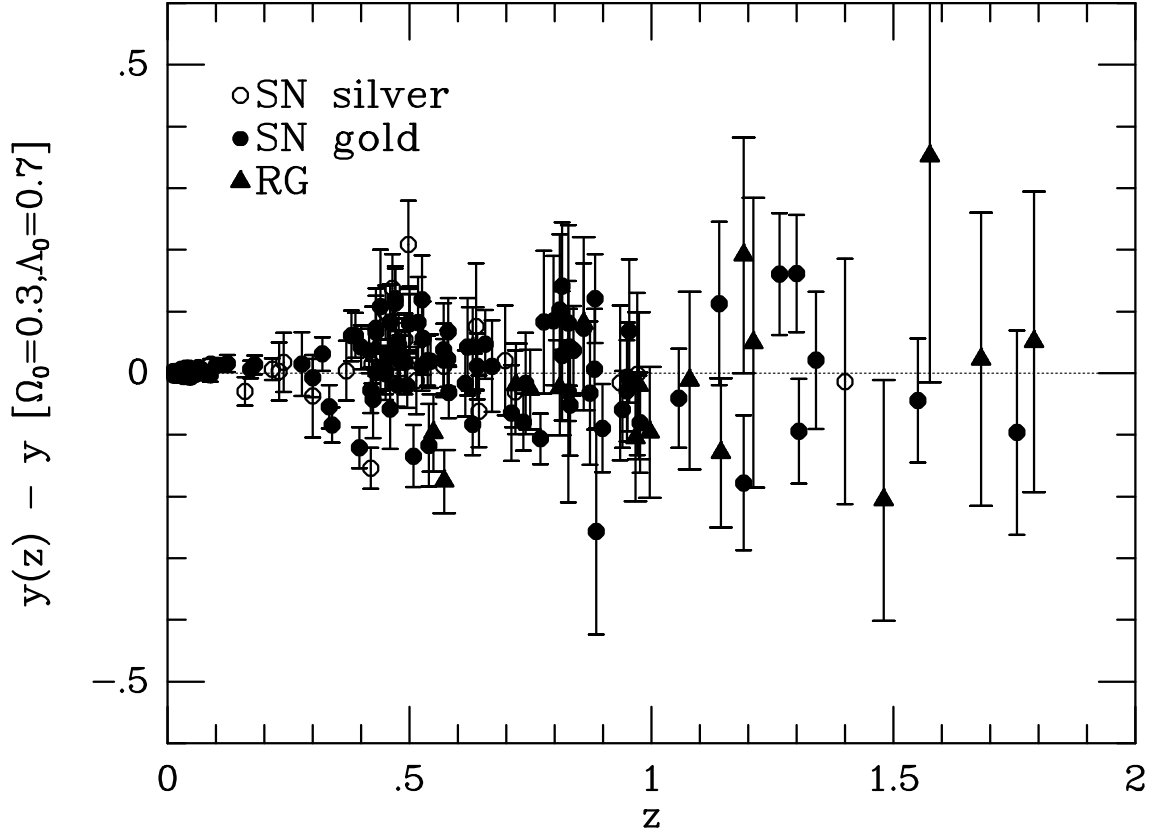


Fig. 1.— The difference between the dimensionless coordinate distances and those expected in a spatially flat universe with a cosmological constant and  $\Omega_0 = 0.3$ . SNe and RGs are plotted with different symbols as indicated. There is no significant systematic offset between them in the redshift range where there is an overlap.

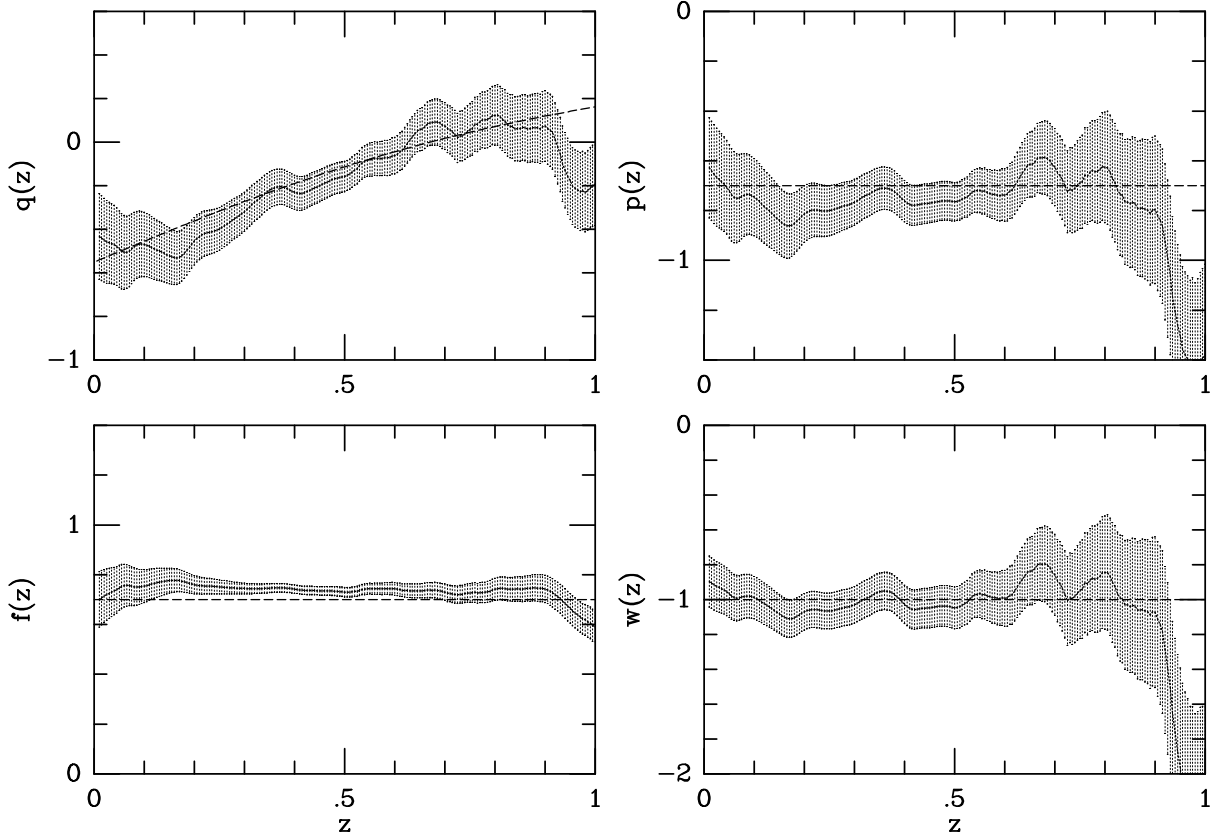


Fig. 2.— Application of our methods on the simulated (pseudo-SNAP) data set, obtained with equations (2), (6), (7), and (8) respectively as described in the text, using a window function with  $\Delta z = 0.4$ . The dotted/hatched regions show the recovered trends for the quantities of interest. The assumed cosmology is a standard Friedmann-Lemaitre model with  $\Omega_0 = 0.3$  and  $\Lambda_0 = 0.7$ , and the theoretical (noiseless) values of the measured quantities are shown as dashed lines. There is a good correspondence (typically well within  $\pm 1\sigma$ ) up to  $z \sim 0.9$ , except in the case of  $f(z)$  where a small systematic bias is present, and the formally evaluated errors may be too small as an artifact of the numerical procedure.

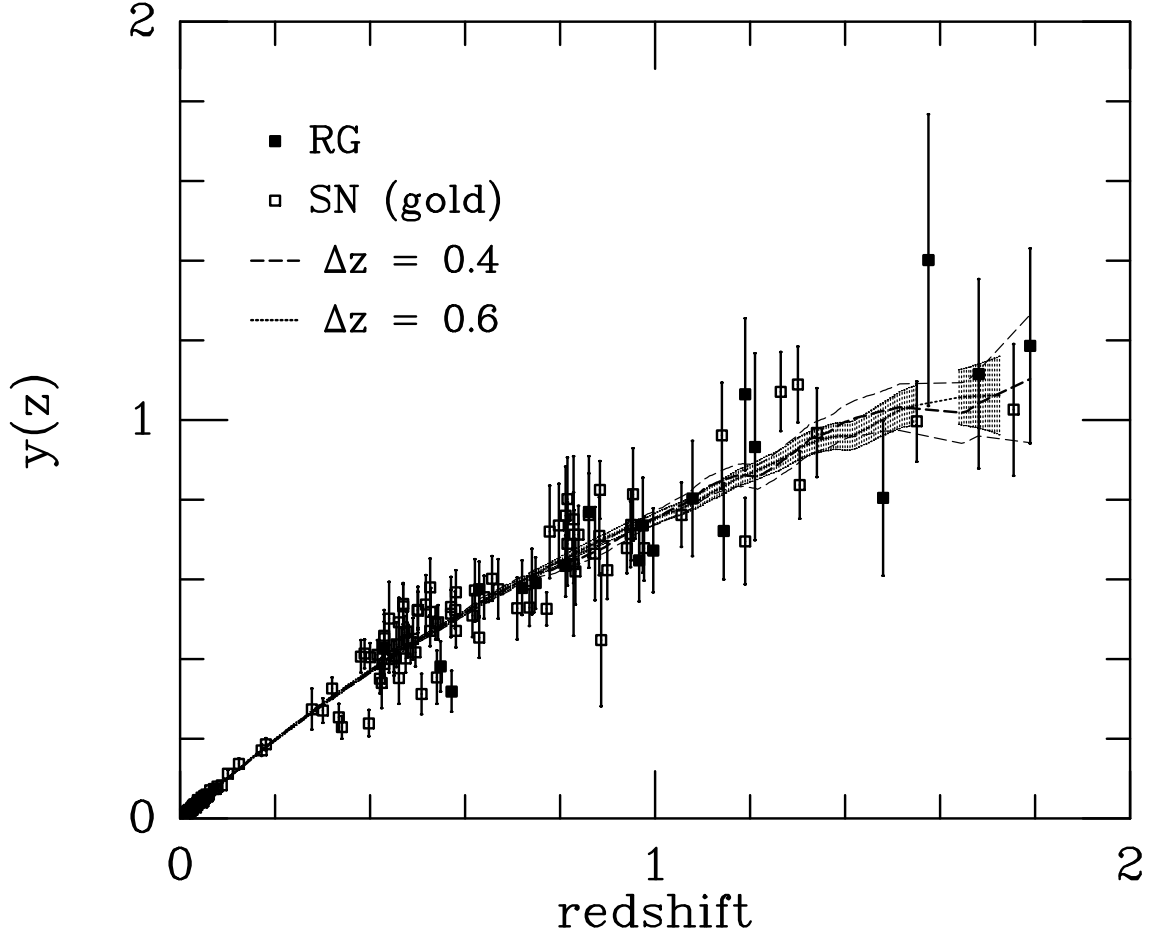


Fig. 3.— Dimensionless coordinate distances  $y(z)$  to 20 radio galaxies and the “gold” sample SNe as a function of  $z$ . The smoothed values of  $y$  along with their  $1\sigma$  error bars obtained for window function widths  $\Delta z = 0.4$  (dashed lines) and  $0.6$  (dotted line and hatched error range) are also shown. Note again that the new high-redshift SNe values agree quite well with those of the high-redshift RGs.

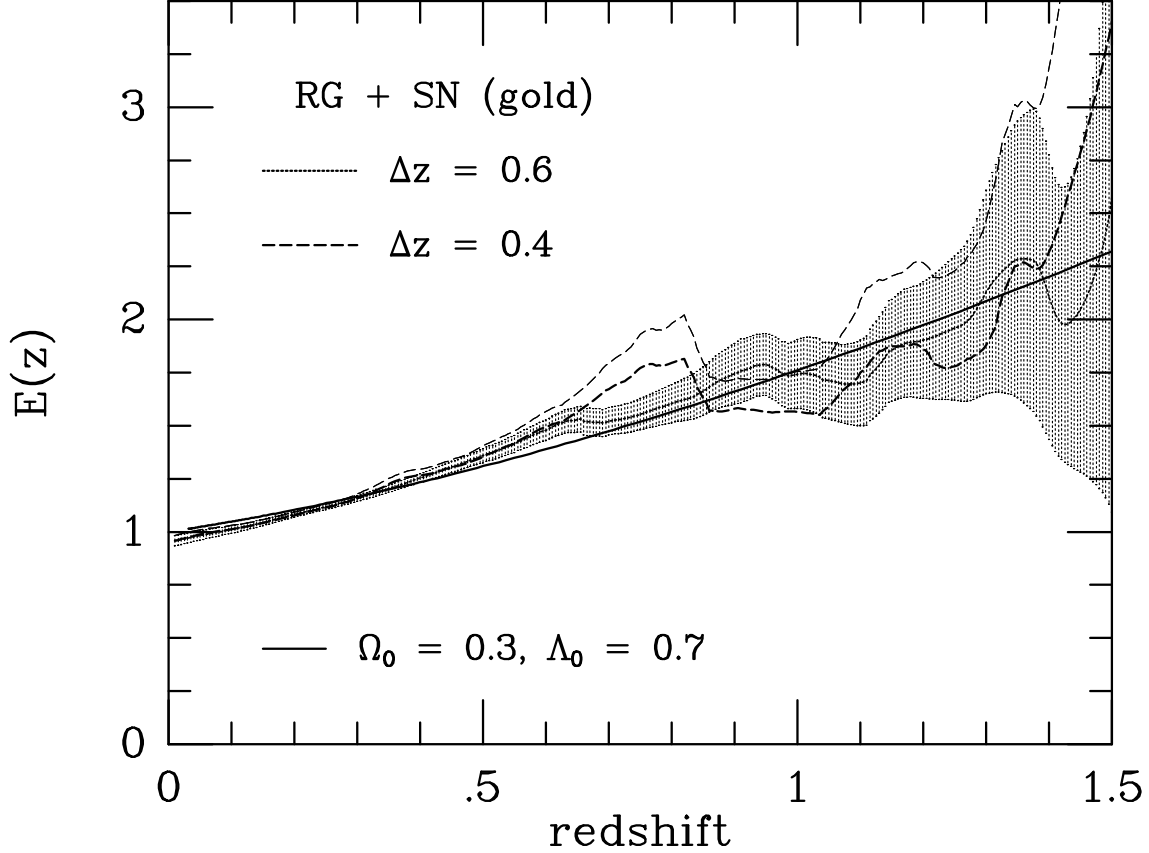


Fig. 4.— The derived values of the dimensionless expansion rate  $E(z) \equiv (\dot{a}/a)H_0^{-1} = (dy/dz)^{-1}$  obtained with window functions of width  $\Delta z = 0.4$  and their  $1\sigma$  error bars (dashed lines) and  $0.6$  (dotted line and hatched error range). At a redshift of zero, the value of  $E$  is  $E_0 = 0.97 \pm 0.03$ . The value of  $E(z)$  predicted in a spatially flat universe with a cosmological constant and  $\Omega_0 = 0.3$  is also shown, and provides a reasonable match to the data.

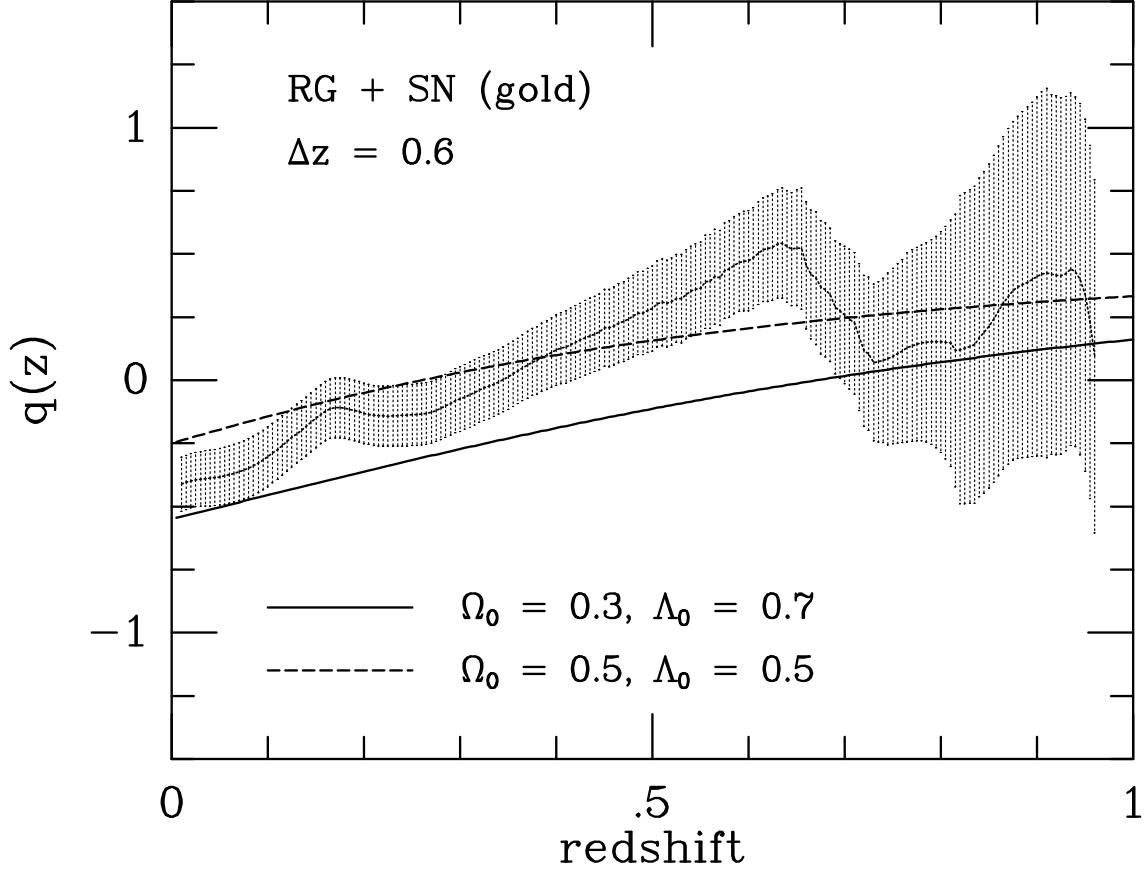


Fig. 5.— The derived values of deceleration parameter  $q(z)$  (see equation 2) and their  $1\sigma$  error bars obtained with window function of width  $\Delta z = 0.6$  applied to the RG plus gold SNe sample. The universe transitions from acceleration to deceleration at a redshift  $z_T \approx 0.4$ . The value of the deceleration parameter at zero redshift is  $q_0 = -0.35 \pm 0.15$ . Note that this determination of  $q(z)$  only depends upon the assumptions that the universe is homogenous, isotropic, expanding, and spatially flat, and it does not depend on any assumptions about the nature of the dark energy, or the correct theory of gravity. Solid and dashed lines show the expected dependence in the standard Friedmann-Lemaître models with zero curvature, for two pairs of values of  $\Omega_0$  and  $\Lambda_0$ .



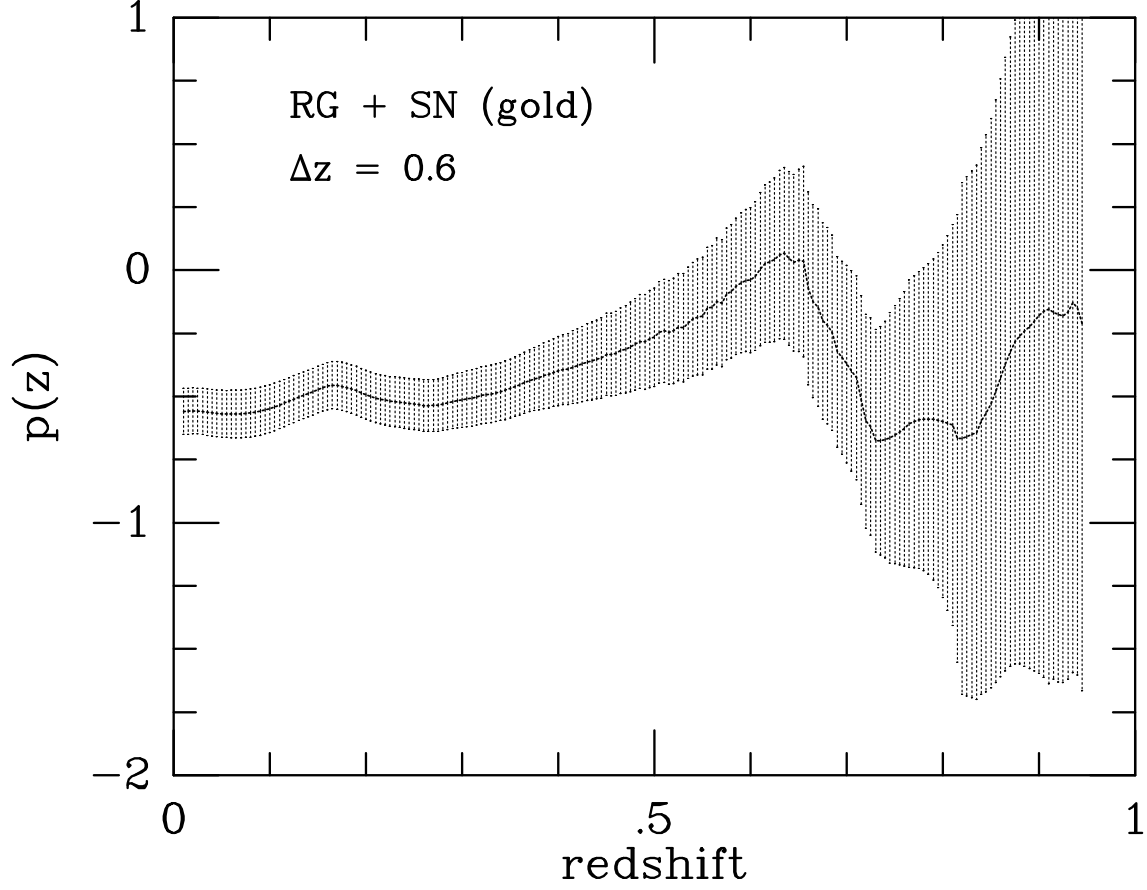


Fig. 6.— The derived values of dark energy pressure  $p(z)$  (see equation 6), obtained with window function of width  $\Delta z = 0.6$ . This derivation of  $p(z)$  requires a choice of theory of gravity, and General Relativity has been adopted here. The value at zero redshift is  $p_0 = -0.6 \pm 0.15$ .

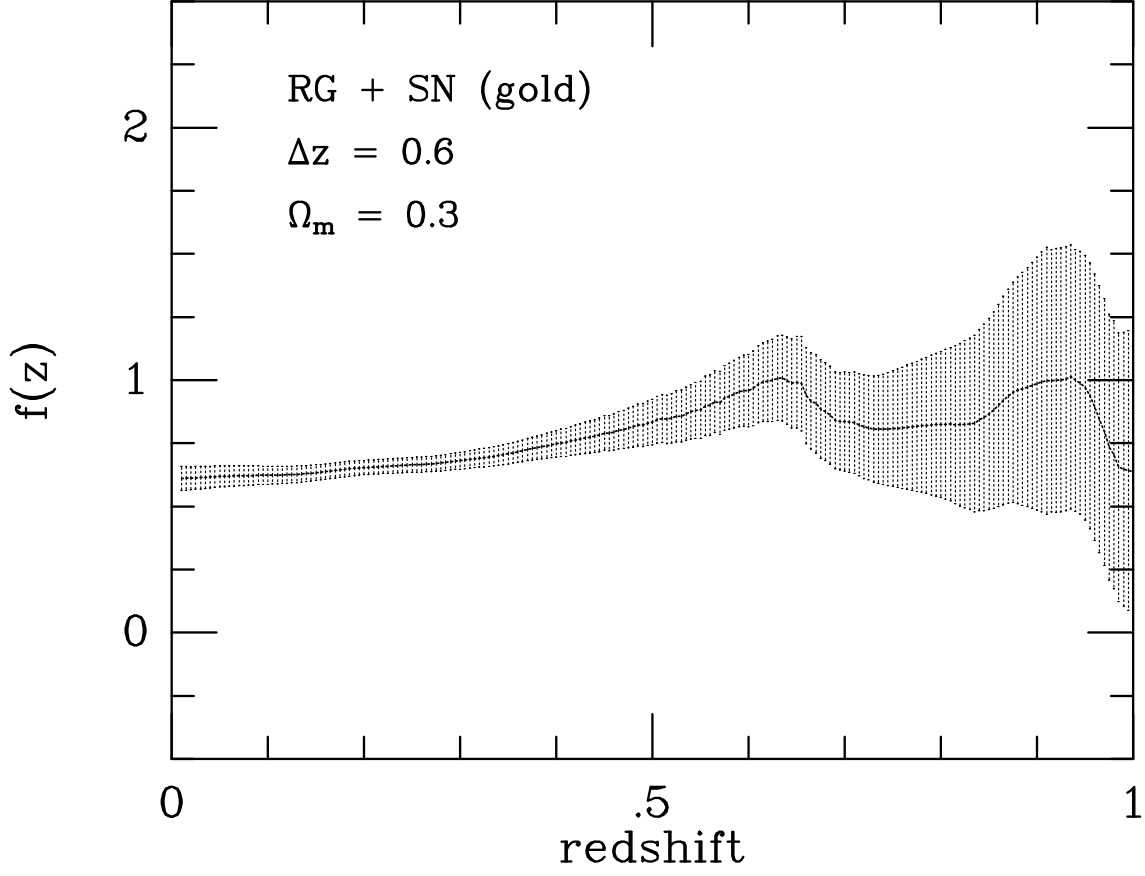


Fig. 7.— The derived values of the dark energy density fraction  $f(z)$  (see equation 7), obtained with window function of width  $\Delta z = 0.6$ . This derivation of  $f(z)$  requires of theory of gravity and the value of  $\Omega_0$  for the nonrelativistic matter; General Relativity has been adopted here, and  $\Omega_0 = 0.3$  is assumed. The value at zero redshift is  $0.62 \pm 0.05$ .

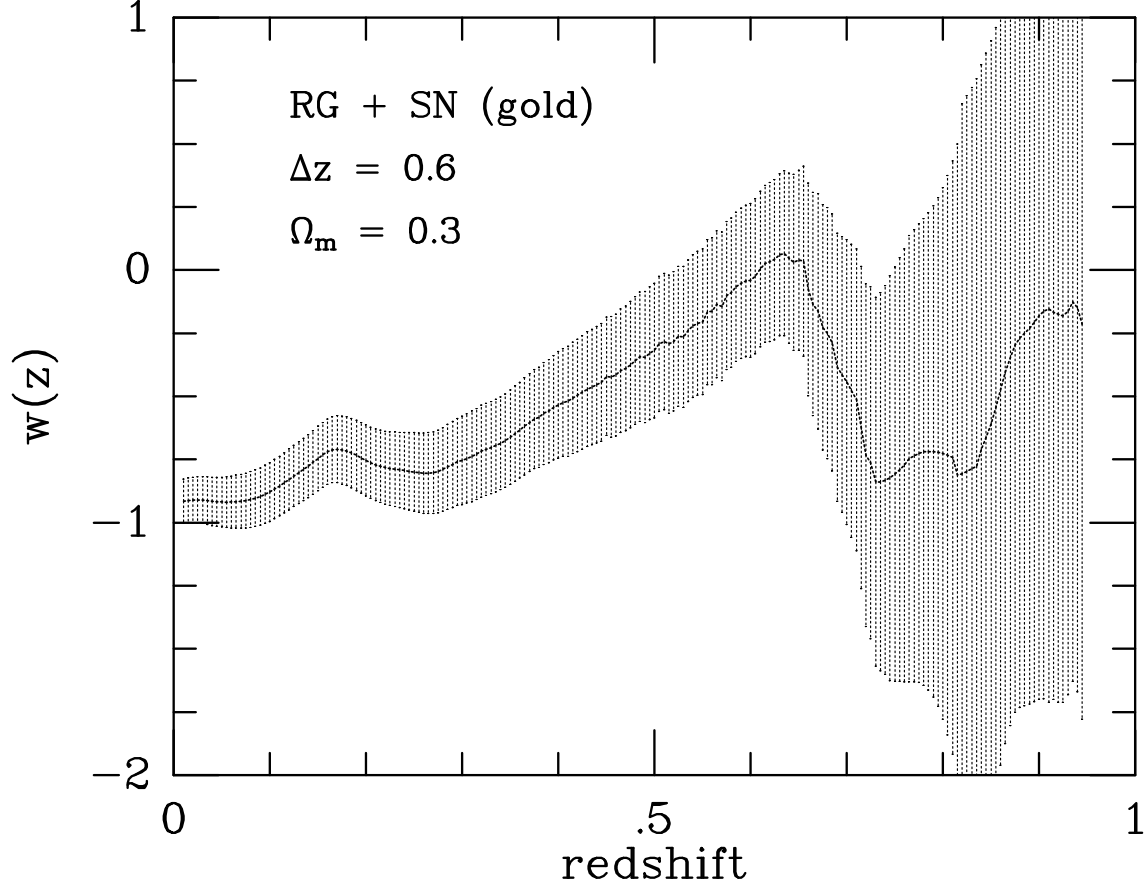


Fig. 8.— The derived values of the dark energy equation of state parameter  $w(z)$  (see equation 8), obtained with window function of width  $\Delta z = 0.6$ . This derivation of  $w(z)$  requires of theory of gravity and the value of  $\Omega_0$ ; General Relativity has been adopted here, and  $\Omega_0 = 0.3$  is assumed. The value at zero redshift is  $w_0 = -0.9 \pm 0.1$ , consistent with the cosmological constant models.